CBCS SCHEME

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21MAT31

Third Semester B.E./B.Tech. Degree Examination, June/July 2025
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the Laplace transform of
$$t^5 e^{4t}\cosh 3t + 5^t + \frac{\sin 2t}{t}$$ **(06 Marks)**

b. Express $f(t) = \begin{cases} \cos t & \text{for} \quad 0 < t \le \pi \\ 1 & \text{for} \quad \pi < t \le 2\pi \\ \sin t & \text{for} \quad \pi > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. **(07 Marks)**

c. Using convolution theorem find the inverse Laplace transform of $\dfrac{1}{s^3(s^2+1)}$. **(07 Marks)**

OR

2 a. Find the inverse Laplace transform of
$$\frac{3s+2}{s^2-s-2}$$ **(06 Marks)**

b. If $f(t) = \begin{cases} t & \text{for} \quad 0 \le t \le \pi \\ 2\pi - t & \text{for} \quad \pi \le t \le 2\pi \end{cases}$ and $f(t+2\pi) = f(t)$ then

show that $L\{f(t)\} = \dfrac{1}{s^2}\tanh\left(\dfrac{\pi s}{2}\right)$ **(07 Marks)**

c. Use Laplace transform to solve $y'' + 6y' + 9y = 12t^2 e^{-3t}$ under the conditions $y(0) = y'(0) = 0$. **(07 Marks)**

Module-2

3 a. Expand
$$f(x) = \frac{\pi - x}{2} \quad \text{in} \quad 0 \le x \le 2\pi$$ as Fourier series expansion. **(06 Marks)**

b. Obtain the Fourier series for the function
$$f(x) = \begin{cases} 1 + \dfrac{4x}{3} & \text{in} \quad -\dfrac{3}{2} < x < 0 \\ 1 - \dfrac{4x}{3} & \text{in} \quad 0 < x < \dfrac{3}{2} \end{cases},$$
hence deduce that $\dfrac{\pi^2}{8} = \dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots\ldots\ldots$ **(07 Marks)**

c. Expand $f(x) = \sin x$ in Fourier half range cosine series over the interval $(0, \pi)$. **(07 Marks)**

OR

4 a. Find the Fourier series expansion of x^2 in $-\pi \le x \le \pi$. **(06 Marks)**

b. Obtain the Fourier half range sine series for the function

$$f(x) = \begin{cases} \frac{1}{4} - x & \text{in } 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{in } \frac{1}{2} < x < 1 \end{cases}$$ **(07 Marks)**

c. A function f(x) of period 2π is specified by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	7.9	7.2	3.6	0.5	0.9	6.8	7.9

Obtain the Fourier series for f(x) upto the first harmonic. **(07 Marks)**

Module-3

5 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1 & \text{in } |x| \le 1 \\ 0 & \text{in } |x| > 1 \end{cases}$$ and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(06 Marks)**

b. Find the Fourier sine and cosine transform of $f(x) = e^{-ax}$ for $a > 0$ and $x > 0$. **(07 Marks)**

c. Obtain the inverse z-transform of $\frac{3z^2 + z}{(5z-1)(5z+2)}$ **(07 Marks)**

OR

6 a. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1 - |x| & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$, hence evaluate $\int_0^\infty \frac{\sin^2 x}{x^2} dx$. **(06 Marks)**

b. Find the z-transform of (i) $\cos n\theta$ (ii) $\sin(n\theta)$ **(07 Marks)**

c. Solve $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ with $u_0 = u_1 = 0$ using z-transform. **(07 Marks)**

Module-4

7 a. Classify the second order partial differential equations :

(i) $u_{xx} + 2u_{xy} + u_{yy} = 0$ (ii) $(x+1)u_{xx} - 2(x+2)u_{xy} + (3+x)u_{yy} = 0$

(iii) $y^2u_{xx} + u_{yy} + u_x^2 + u_y^2 + 7 = 0$ (iv) $(1+x^2)u_{xx} + (5+2x^2)u_{xt} + (4+x^2)u_{tt} = 0$ **(10 Marks)**

b. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ [Laplace equation] for the following square mesh with boundary values as shown in Fig.Q7(b). Use Leibamann's method for 1[st] iteration.

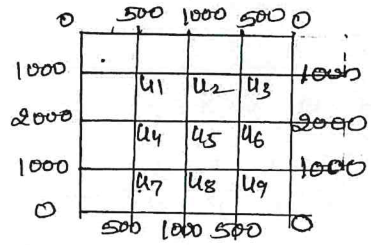

Fig.Q7(b) **(10 Marks)**

OR

8 a. Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2},$$ subject to $u(0, t) = u(4, t) = 0$, $\frac{\partial u}{\partial t}(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ by taking step length in x, h = 1. **(10 Marks)**

b. Solve $u_t = u_{xx}$ subject to the conditions $u(0, t) = u(1, t) = 0$, $u(x, 0) = \sin(\pi x)$, $0 \le t \le 0.1$ by taking h = 0.2, by applying Bendre-Schemidt explicit formula, hence find
(i) u(0.2, 0.04) (ii) u(0.6, 0.06) **(10 Marks)**

Module-5

9 a. By fourth order Runge-Kutta method, solve

$$\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$$ for x = 0.2, correct to four decimal places using the initial conditions $y = 1$ and $\frac{dy}{dx} = 0$, when x = 0. **(06 Marks)**

b. Derive Euler's equation in the standard form

$$\frac{\partial f}{\partial y} - \frac{d\left(\frac{\partial f}{\partial y'}\right)}{dx} = 0$$ **(07 Marks)**

c. Show that the equation of the curve joining the points (0, 1) and (1, 2) for which

$$I = \int_0^1 \sqrt{1 + (y')^2}\, dx$$ is extremum, is a straight line. **(07 Marks)**

OR

10 a. Apply Milne's method to find y(0.4) from the $y'' + xy' + y = 0$ and initial values as $y(0) = 1$, $y(0.1) = 0.995$, $y(0.2) = 0.9801$, $y(0.3) = 0.956$, $y'(0) = 0$, $y'(0.1) = -0.0995$, $y'(0.2) = -0.196$, $y'(0.3) = -0.2867$. **(07 Marks)**

b. Prove that geodesics on a plane are straight line. **(06 Marks)**

c. Find the curve on which the functional $\int_0^{\pi/2} \left[(y')^2 - y^2 + 2xy\right] dx$ with $y(0) = y(\pi/2) = 0$ can be extremised. **(07 Marks)**

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